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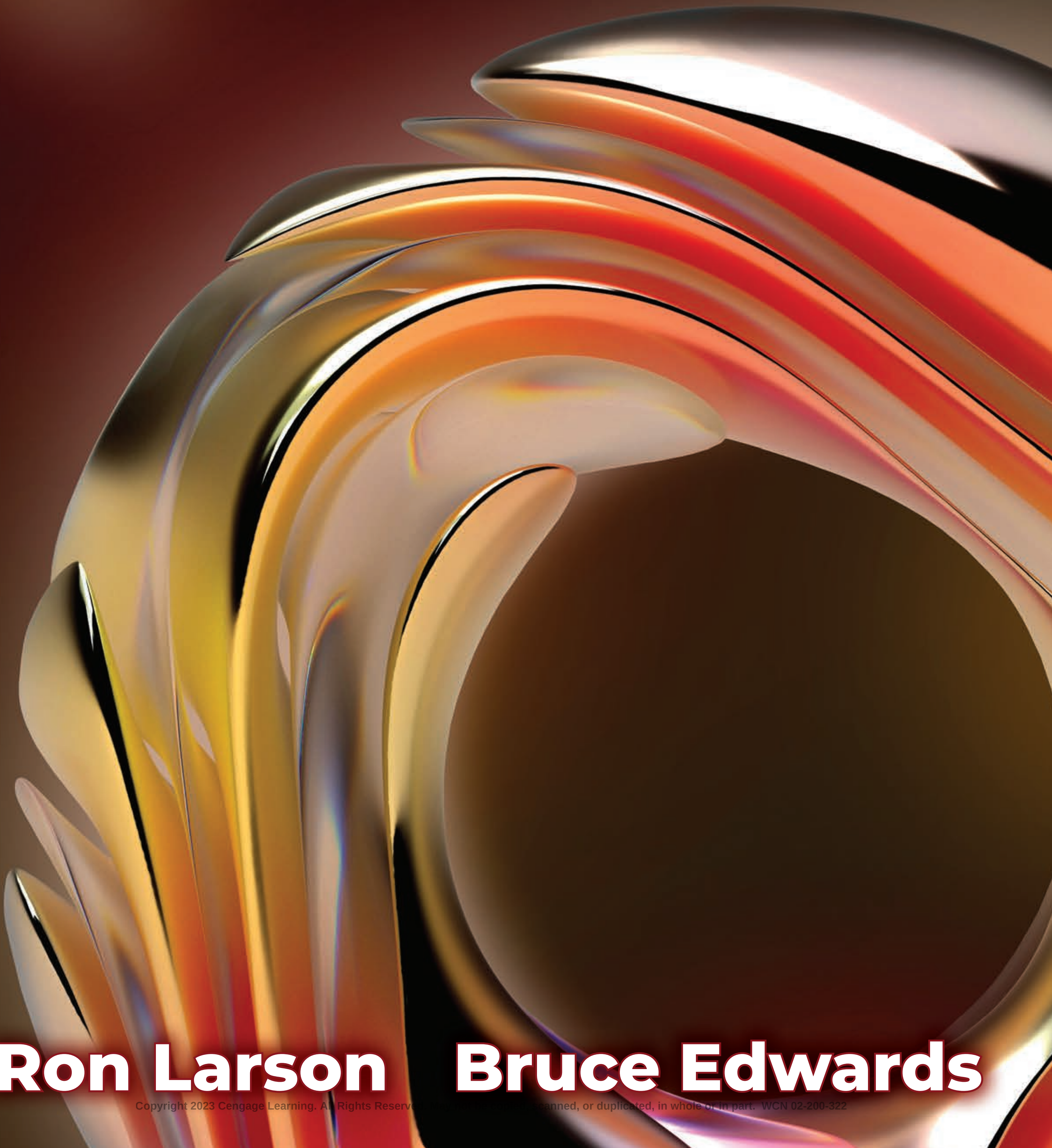


Ron Larson Bruce Edwards

Index of Applications

Engineering and Physical Sciences

(continued on back inside cover)

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
22. $\frac{d}{dx}[\text{arccot}\, u] = \frac{-u'}{1 + u^2}$
23. $\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
24. $\frac{d}{dx}[\text{arccsc}\, u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27. $\frac{d}{dx}[\tanh u] = (\text{sech}^2\, u)u'$
28. $\frac{d}{dx}[\coth u] = -(\text{csch}^2\, u)u'$
29. $\frac{d}{dx}[\text{sech}\, u] = -(\text{sech}\, u \tanh u)u'$
30. $\frac{d}{dx}[\text{csch}\, u] = -(\text{csch}\, u \coth u)u'$
31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2 + 1}}$
32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2 - 1}}$
33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1 - u^2}$
34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1 - u^2}$
35. $\frac{d}{dx}[\text{sech}^{-1}\, u] = \frac{-u'}{u\sqrt{1 - u^2}}$
36. $\frac{d}{dx}[\text{csch}^{-1}\, u] = \frac{-u'}{|u|\sqrt{1 + u^2}}$

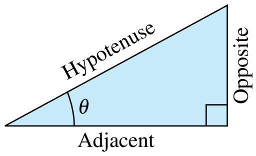
Basic Integration Formulas

1. $\int kf(u)\, du = k\int f(u)\, du$
2. $\int [f(u) \pm g(u)]\, du = \int f(u)\, du \pm \int g(u)\, du$
3. $\int du = u + C$
4. $\int u^n\, du = \frac{u^{n+1}}{n + 1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u\, du = e^u + C$
7. $\int a^u\, du = \left(\frac{1}{\ln a}\right)a^u + C$
8. $\int \sin u\, du = -\cos u + C$
9. $\int \cos u\, du = \sin u + C$
10. $\int \tan u\, du = -\ln|\cos u| + C$
11. $\int \cot u\, du = \ln|\sin u| + C$
12. $\int \sec u\, du = \ln|\sec u + \tan u| + C$
13. $\int \csc u\, du = -\ln|\csc u + \cot u| + C$
14. $\int \sec^2 u\, du = \tan u + C$
15. $\int \csc^2 u\, du = -\cot u + C$
16. $\int \sec u \tan u\, du = \sec u + C$
17. $\int \csc u \cot u\, du = -\csc u + C$
18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \text{arcsec} \frac{|u|}{a} + C$

TRIGONOMETRY

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

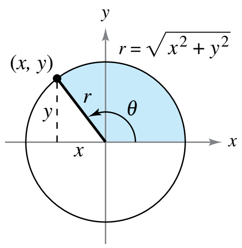


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

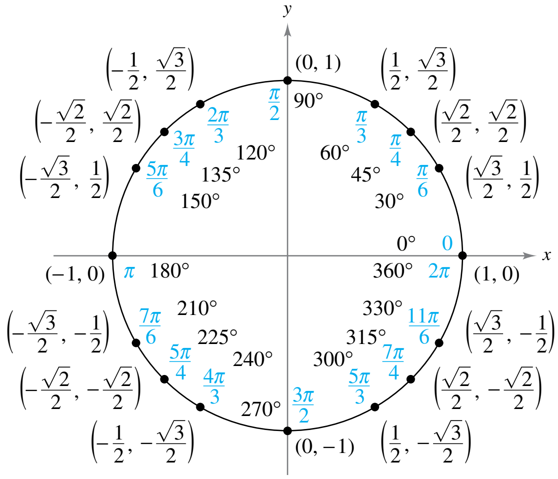
Circular function definitions, where θ *is any angle.*



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \qquad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Even/Odd Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x$$

$$\csc(-x) = -\csc x \qquad \tan(-x) = -\tan x$$

$$\sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u + v}{2}\right)\cos\left(\frac{u - v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u + v}{2}\right)\sin\left(\frac{u - v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u + v}{2}\right)\cos\left(\frac{u - v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u + v}{2}\right)\sin\left(\frac{u - v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

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*Available at the text companion website *LarsonCalculus.com*

Preface

Welcome to *Calculus* with CalcChat® and CalcView®, Twelfth Edition. We are excited to offer you a new edition with more resources then ever that will help you understand and master calculus. This text includes features and resources that continue to make *Calculus* a valuable learning tool for students and a trustworthy teaching tool for instructors.

Calculus provides the clear instruction, precise mathematics, and thorough coverage that you expect for your course. Additionally, this new edition provides you with **free** access to a variety of digital resources.

- **GO DIGITAL**—direct access to digital content on your mobile device or computer
- **CalcView.com**—video solutions to selected exercises
- **CalcChat.com**—worked-out solutions to odd-numbered exercises and access to online tutors
- **LarsonCalculus.com**—companion website with resources to supplement your learning

These digital resources will help enhance and reinforce your understanding of the material presented in this text and prepare you for future mathematics courses. CalcView® and CalcChat® are also available as free mobile apps.

Features

NEW GO DIGITAL

Scan the on-page codes of this text to *GO DIGITAL* on your mobile device. This will give you easy access to

- instructional and proof videos,
- interactive examples,
- solutions to exercises,
- free online tutoring,

and many other resources.

UPDATED CalcView®

The website *CalcView.com* provides video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. The CalcView mobile app is available for free at the Apple® App Store® or Google Play™ store. You can access the video solutions by scanning the on-page codes at the beginning of the section exercises or visiting the *CalcView.com* website.

UPDATED CalcChat®

Solutions to all odd-numbered exercises are provided for free at *CalcChat.com.* Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. **For over 20 years, millions of students have visited our site for help!** The CalcChat mobile app is also available as a free download at the Apple® App Store® or Google Play™ store.

UPDATED LarsonCalculus.com

All companion website features have been updated based on this revision. Watch videos explaining concepts or proofs from the text, explore examples, view three-dimensional graphs, download articles from math journals, and much more.

App Store is a service mark of Apple Inc. Google Play is a trademark of Google Inc.

NEW Big Ideas of Calculus

We have added a new feature to help you discover and understand the Big Ideas of Calculus. This feature, which is denoted by [icon], has four parts.

- The **Big Ideas of Calculus** notes give you an overview of the major concepts of a chapter and how they relate to the earlier concepts you have studied. These notes appear near the beginning of a chapter and in the chapter review.
- In each section and in the chapter review, make sure you do the **Concept Check** exercises and the **Exploring Concepts** exercises. These exercises will help you develop a deeper and clearer knowledge of calculus. Work through these exercises to build and strengthen your understanding of the concepts.
- To continue exploring calculus, do the **Building on Concepts** exercises at the end of the chapter review. Not only will these exercises help you expand your knowledge and use of calculus, they will prepare you to learn concepts in later chapters.

Big Ideas of Calculus

In this chapter, you will study integration. Integration, like differentiation, is a major theme of calculus. Interestingly, as you will learn in the Fundamental Theorem of Calculus, these two major themes have an inverse relationship. As part of your study of integration, you will examine the areas of plane regions.

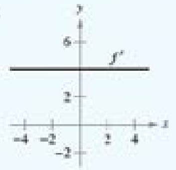
Concept Check

1. Explain what it means for a function F to be an antiderivative of a function f on an interval I.
2. Can two different functions both be antiderivatives of the same function? Explain.
3. Explain how to find a particular solution of a ... equation.
4. Describe the difference between the general s... particular solution of a differential equation.

Exploring Concepts

In Exercises 49 and 50, the graph of the derivative of a function is given. Sketch the graphs of *two* functions that have the given derivative. (There is more than one correct answer.) To print an enlarged copy of the graph, go to *MathGraphs.com*.

49.



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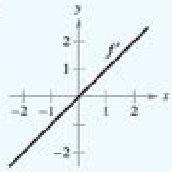


Building on Concepts

75. Consider the function

$$F(x) = \int_0^x \sin^2 t\, dt.$$

(a) Evaluate F at $x = 0, \pi/6, \pi/3, \pi/2, 2\pi/3, 5\pi/6$, and π. Are the values of F increasing or decreasing? Explain.

(b) Use the integration capabilities of a graphing utility to graph F and $y_1 = \sin^2 t$ on the interval $0 \le t \le \pi$.

(c) Use the differentiation capabilities of a graphing utility to graph F'. How is this graph related to the graph in part (b)?

(d) Verify that $\sin^2 t$ is the derivative of

$$y = \frac{1}{2}t - \frac{1}{4}\sin 2t.$$

Graph y and write a short paragraph about how this graph is related to those in parts (b) and (c).

UPDATED Exercise Sets

The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant and to include topics our users have suggested. The exercises are organized and titled so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations.

SECTION PROJECT

Graphs and Limits of Trigonometric Functions

Recall from Theorem 1.9 that the limit of

$$f(x) = \frac{\sin x}{x}$$

as x approaches 0 is 1.

(a) Use a graphing utility to graph the function f on the interval $-\pi \le x \le \pi$. Explain how the graph helps confirm this theorem.

(b) Explain how you could use a table of values to confirm the value of this limit numerically.

(c) Graph $g(x) = \sin x$ by hand. Sketch a tangent line at the point $(0, 0)$ and visually estimate the slope of this tangent line.

(d) Let $(x, \sin x)$ be a point on the graph of g near $(0, 0)$, and write a formula for the slope of the secant line joining $(x, \sin x)$ and $(0, 0)$. Evaluate this formula at $x = 0.1$ and $x = 0.01$. Then find the exact slope of the tangent line to g at the point $(0, 0)$.

(e) Sketch the graph of the cosine function $h(x) = \cos x$. What is the slope of the tangent line at the point $(0, 1)$? Use limits to find this slope analytically.

(f) Find the slope of the tangent line to $k(x) = \tan x$ at $(0, 0)$.

Section Projects

Projects appear in selected sections and encourage you to explore applications related to the topics you are studying. All of these projects provide an interesting and engaging way for you and other students to work and investigate ideas collaboratively.

UPDATED Chapter Opener

Each Chapter Opener highlights real-life applications used in the examples and exercises. For this edition, we also highlight the online resources at *CalcView.com* and *CalcChat.com*.

Section Objectives

A bulleted list of learning objectives provides you with the opportunity to preview what will be presented in the upcoming section.

Theorems

Theorems provide the conceptual framework for calculus. Theorems are clearly stated and separated from the rest of the text by boxes for quick visual reference. Key proofs often follow the theorem and can be found at *LarsonCalculus.com.*

Definitions

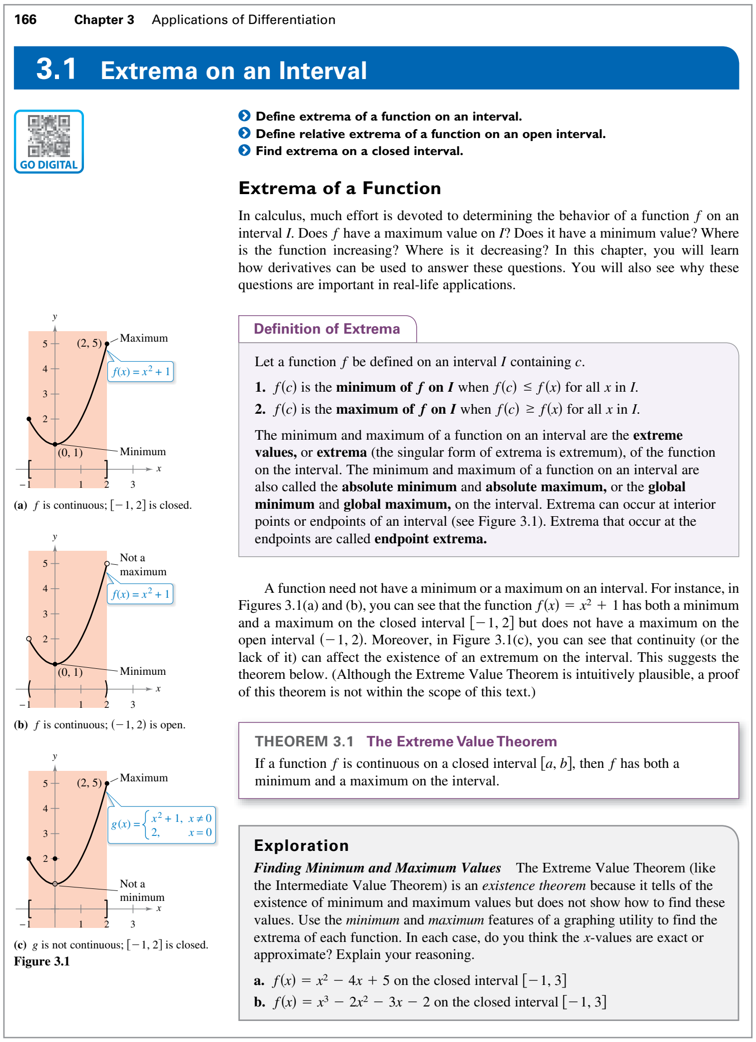
As with theorems, definitions are clearly stated using precise, formal wording and are separated from the text by boxes for quick visual reference.

Explorations

Explorations provide unique challenges to study concepts that have not yet been formally covered in the text. They allow you to learn by discovery and introduce topics related to ones presently being studied. Exploring topics in this way encourages you to think outside the box.

UPDATED Remarks

These hints and tips reinforce or expand upon concepts, help you learn how to study mathematics, caution you about common errors, address special cases, or show an alternative solution to an example. We have added several new Remarks to help students who need more in-depth algebra support.



166 Chapter 3 Applications of Differentiation

3.1 Extrema on an Interval

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- Define extrema of a function on an interval.
- Define relative extrema of a function on an open interval.
- Find extrema on a closed interval.

Extrema of a Function

In calculus, much effort is devoted to determining the behavior of a function f on an interval I. Does f have a maximum value on I? Does it have a minimum value? Where is the function increasing? Where is it decreasing? In this chapter, you will learn how derivatives can be used to answer these questions. You will also see why these questions are important in real-life applications.

Definition of Extrema

Let a function f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of f on I** when $f(c) \le f(x)$ for all x in I.
2. $f(c)$ is the **maximum of f on I** when $f(c) \ge f(x)$ for all x in I.

The minimum and maximum of a function on an interval are the **extreme values,** or **extrema** (the singular form of extrema is extremum), of the function on the interval. The minimum and maximum of a function on an interval are also called the **absolute minimum** and **absolute maximum,** or the **global minimum** and **global maximum,** on the interval. Extrema can occur at interior points or endpoints of an interval (see Figure 3.1). Extrema that occur at the endpoints are called **endpoint extrema.**

(a) f is continuous; $[-1, 2]$ is closed.
(b) f is continuous; $(-1, 2)$ is open.
(c) g is not continuous; $[-1, 2]$ is closed.
Figure 3.1

A function need not have a minimum or a maximum on an interval. For instance, in Figures 3.1(a) and (b), you can see that the function $f(x) = x^2 + 1$ has both a minimum and a maximum on the closed interval $[-1, 2]$ but does not have a maximum on the open interval $(-1, 2)$. Moreover, in Figure 3.1(c), you can see that continuity (or the lack of it) can affect the existence of an extremum on the interval. This suggests the theorem below. (Although the Extreme Value Theorem is intuitively plausible, a proof of this theorem is not within the scope of this text.)

THEOREM 3.1 The Extreme Value Theorem

If a function f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

Exploration

Finding Minimum and Maximum Values The Extreme Value Theorem (like the Intermediate Value Theorem) is an *existence theorem* because it tells of the existence of minimum and maximum values but does not show how to find these values. Use the *minimum* and *maximum* features of a graphing utility to find the extrema of each function. In each case, do you think the x-values are exact or approximate? Explain your reasoning.

a. $f(x) = x^2 - 4x + 5$ on the closed interval $[-1, 3]$
b. $f(x) = x^3 - 2x^2 - 3x - 2$ on the closed interval $[-1, 3]$

UPDATED Historical Notes and Biographies

Historical Notes provide you with background information on the foundations of calculus. The Biographies introduce you to the people who created and contributed to calculus. We have added several new biographies, and more biographies are available at *LarsonCalculus.com*.

Technology

Throughout the book, technology boxes show you how to use technology to solve problems and explore concepts of calculus. These tips also point out some pitfalls of using technology.

How Do You See It? Exercise

The How Do You See It? exercise in each section presents a problem that you will solve by visual inspection using the concepts learned in the lesson.

UPDATED Applications

Carefully chosen applied exercises and examples are included throughout to address the question, "When will I use this?" These applications are pulled from diverse sources, such as current events, world data, industry trends, and more, and relate to a wide range of interests. Understanding where calculus is (or can be) used promotes fuller understanding of the material.

Putnam Exam Challenges

Putnam Exam questions appear in selected sections. These actual Putnam Exam questions will challenge you and push the limits of your understanding of calculus.

Student Resources

CENGAGE | WEBASSIGN

Prepare for class with confidence using WebAssign from Cengage. This online learning platform, which includes an interactive eBook, fuels practice so that you truly absorb what you learn and prepare better for tests. Videos and tutorials walk you through concepts and deliver instant feedback and grading, so you always know where you stand in class. Focus your study time and get extra practice where you need it most. Study smarter with WebAssign! Ask your instructor today how you can get access to WebAssign, or learn about self-study options at *cengage.com/webassign*.

Student Solutions Manual

Student Solutions Manual for Calculus of a Single Variable, 12e
(ISBN-13: 978-0-357-74919-7)
Student Solutions Manual for Multivariable Calculus, 12e
(ISBN-13: 978-0-357-74920-3)
These manuals provide step-by-step solutions for all odd-numbered exercises, including Review Exercises and P.S. Problem Solving. The manual for *Calculus of a Single Variable* contains solutions for Chapters P–10, and the manual for *Multivariable Calculus* contains solutions for Chapters 11–16.

Cengage.com

Additional student resources for this product are available online. Sign up or sign in at *cengage.com* to search for and access this product and its online resources.

LarsonCalculus.com

Of the many features at this website, students have told us that the videos are the most helpful. Watch instructional videos presented by Dana Mosely, as he explains various calculus concepts. Watch proof videos presented by Bruce Edwards, as he explains various calculus theorems and their proofs. Other helpful features are the data downloads (editable spreadsheets so you do not have to enter the data), algebra help videos, interactive examples, and much more. You can access these features by going to *LarsonCalculus.com* or by scanning the on-page code.

CalcChat.com

This website provides free step-by-step solutions to all odd-numbered exercises and tests. Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. You can access the solutions by going to *CalcChat.com* or by scanning the on-page code on the first page of any exercise set.

CalcView.com

This website has free video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. You can access the videos by going to *CalcView.com* or by scanning the on-page code on the first page of the section exercises.

MathGraphs.com

For exercises that ask you to draw on the graph, we have provided free, printable graphs at *MathGraphs.com*. You can access the printable graphs by going to *MathGraphs.com* or by scanning the on-page code on the first page of any exercise set.

Instructor Resources

CENGAGE | WEBASSIGN

Built by educators, WebAssign from Cengage is a fully customizable online solution for STEM disciplines. WebAssign includes the flexibility, tools, and content you need to create engaging learning experiences for your students. The patented grading engine provides unparalleled answer evaluation, giving students instant feedback, and insightful analytics highlight exactly where students are struggling. For more information, visit *cengage.com/webassign*.

Cengage.com

Additional instructor resources for this product are available online. Instructor assets include an Instructor's Manual, Educator's Guide, PowerPoint® slides, a Solution and Answer Guide, and a test bank powered by Cognero®. Sign up or sign in at *cengage.com* to search for and access this product and its online resources. The Cengage Instructor Center is an all-in-one resource for class preparation, presentation, and testing. The instructor resources available for download include:

Instructor's Manual Includes activities and assessments correlated by learning objectives, chapter and section outline, key formulas and terms with definitions, ideas for student collaboration and class discussions, and more.

Solution and Answer Guide Provides answers and solutions to all exercises, including Review Exercises, P.S. Problem Solving, and Putnam Exam Challenge.

Cengage Testing Powered by Cognero® A flexible online system that allows you to author, edit, and manage test bank content online. You can create multiple tests in an instant and deliver them from your LMS, or export to printable PDF or Word format for in-class assessment.

PowerPoint® Slides The PowerPoint® slides are ready-to-use, visual outlines of each section that can be easily customized for your lectures. Presentations include activities, examples, and ample opportunities for student engagement and interaction.

Transition Guide Highlights the content changes from the previous edition to the new edition, including exercise correlations.

Guide to Online Teaching Provides technological and pedagogical considerations and tips for teaching a calculus course online.

Educator's Guide Offers suggested content and activities for Cengage WebAssign—like videos and assignments—that you can integrate into your course to help boost engagement and outcomes.

LarsonCalculus.com

In addition to its student resources, *LarsonCalculus.com* also has resources to help instructors. For students who need algebra help, we have provided instructional videos to explain various algebra and precalculus concepts. Students can assess their knowledge of these concepts through self-grading progress checks. You can also give your students experience using an online graphing utility with the Interactive Examples. You can access these features by going to *LarsonCalculus.com* or by scanning the on-page code.

MathArticles.com

This text contains over 50 references to articles from mathematics journals noted in the For Further Information feature. To make the articles easily accessible to instructors and students, they are available at *MathArticles.com* or by scanning the on-page code.

Acknowledgments

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Reviewers

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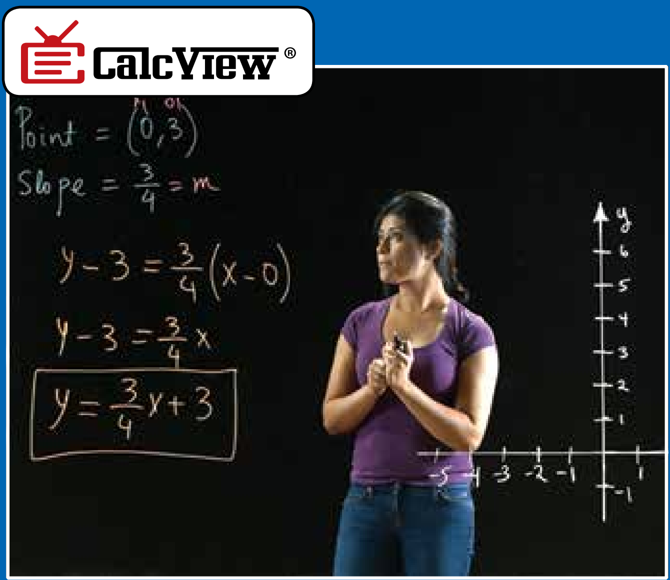
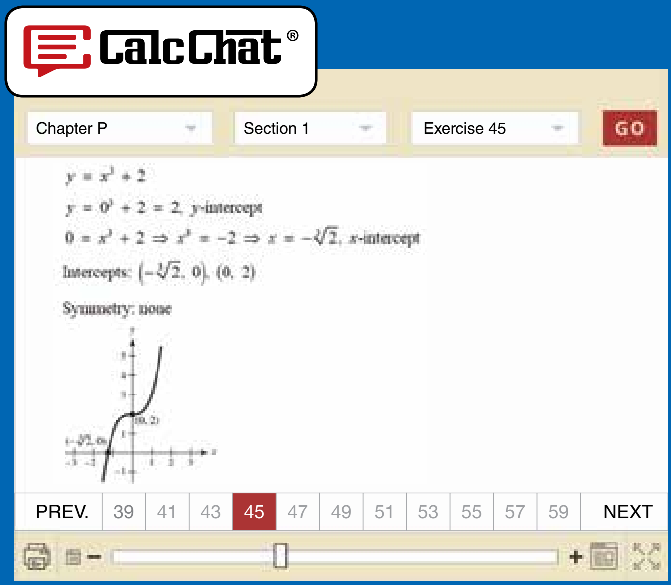
If you have suggestions for improving this text, please feel free to write to us. Over the years we have received many useful comments from both instructors and students, and we value these very much.

Ron Larson

Bruce Edwards

P Preparation for Calculus





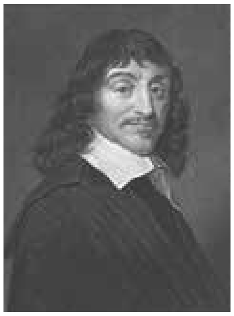
P.3 Automobile Aerodynamics *(Exercise 101, p. 30)*

P.1 Modeling Carbon Dioxide Concentration *(Example 6, p. 7)*

P.1 Graphs and Models

GO DIGITAL

- **Sketch the graph of an equation.**
- **Find the intercepts of a graph.**
- **Test a graph for symmetry with respect to an axis and the origin.**
- **Find the points of intersection of two graphs.**
- **Interpret mathematical models for real-life data.**



RENÉ DESCARTES (1596–1650)

Descartes made many contributions to philosophy, science, and mathematics. The idea of representing points in the plane by pairs of real numbers and representing curves in the plane by equations was described by Descartes in his book *La Géométrie*, published in 1637.

See LarsonCalculus.com to read more of this biography.

The Graph of an Equation

In 1637, the French mathematician René Descartes revolutionized the study of mathematics by combining its two major fields—algebra and geometry. With Descartes's coordinate plane, geometric concepts could be formulated analytically and algebraic concepts could be viewed graphically. The power of this approach was such that within a century of its introduction, much of calculus had been developed.

The same approach can be followed in your study of calculus. That is, by viewing calculus from multiple perspectives—*graphically*, *analytically*, and *numerically*—you will increase your understanding of core concepts.

Consider the equation $3x + y = 7$. The point $(2, 1)$ is a **solution point** of the equation because the equation is satisfied (is true) when 2 is substituted for x and 1 is substituted for y. This equation has many other solutions, such as $(1, 4)$ and $(0, 7)$. To find other solutions systematically, solve the original equation for y.

$$y = 7 - 3x$$ Analytic approach

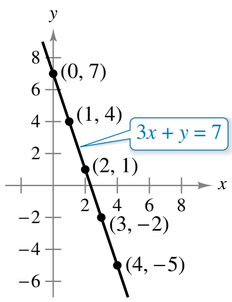
Then construct a **table of values** by substituting several values of x.

x	0	1	2	3	4
y	7	4	1	-2	-5

Numerical approach

From the table, note that $(0, 7)$, $(1, 4)$, $(2, 1)$, $(3, -2)$, and $(4, -5)$ are solutions of the original equation $3x + y = 7$. Like many equations, this equation has an infinite number of solutions. The set of all solution points is the **graph** of the equation, as shown in Figure P.1. Note that the sketch shown in Figure P.1 is referred to as the graph of $3x + y = 7$, even though it really represents only a *portion* of the graph. The entire graph would extend beyond the page.

In this course, you will study many sketching techniques. The simplest is point plotting—that is, you plot points until the basic shape of the graph seems apparent.

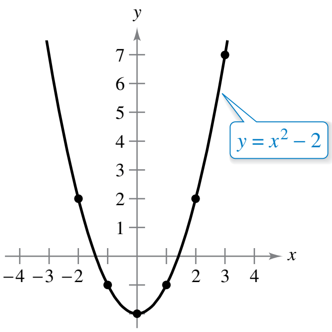


Graphical approach: $3x + y = 7$
Figure P.1

EXAMPLE 1 Sketching a Graph by Point Plotting

To sketch the graph of $y = x^2 - 2$, first construct a table of values. Next, plot the points shown in the table. Then connect the points with a smooth curve, as shown in Figure P.2. This graph is a **parabola.** It is one of the conics you will study in Chapter 10.

x	-2	-1	0	1	2	3
y	2	-1	-2	-1	2	7



The parabola $y = x^2 - 2$
Figure P.2

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One disadvantage of point plotting is that to get a good idea about the shape of a graph, you may need to plot many points. With only a few points, you could badly misrepresent the graph. For instance, to sketch the graph of

$$y = \frac{1}{30}x(39 - 10x^2 + x^4)$$

you plot five points:

$$(-3, -3), \quad (-1, -1), \quad (0, 0), \quad (1, 1), \quad \text{and} \quad (3, 3)$$

as shown in Figure P.3(a). From these five points, you might conclude that the graph is a line. This, however, is not correct. By plotting several more points, you can see that the graph is more complicated, as shown in Figure P.3(b).

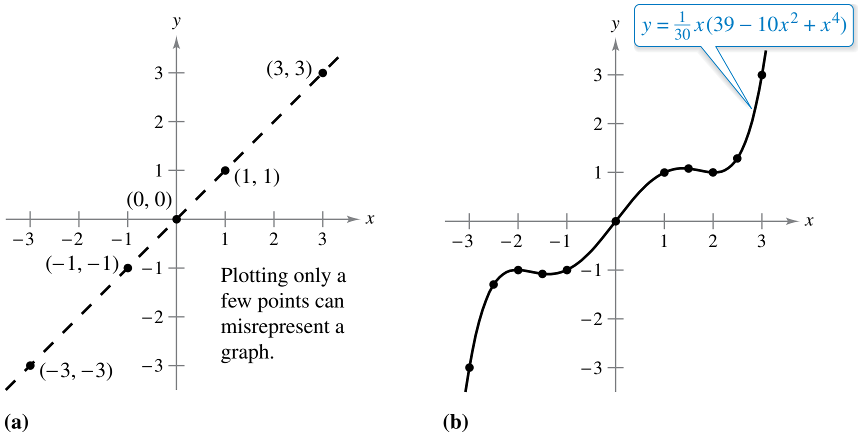
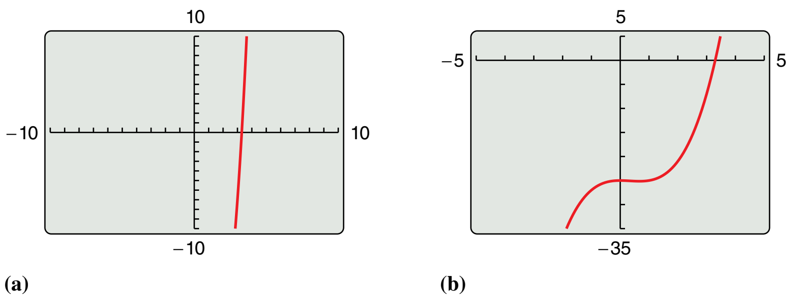


Figure P.3

TECHNOLOGY Graphing an equation has been made easier by technology. Even with technology, however, it is possible to misrepresent a graph badly. For instance, each graphing utility* screen in Figure P.4 shows a portion of the graph of

$$y = x^3 - x^2 - 25.$$

From Figure P.4(a), you might assume that the graph is a line. From Figure P.4(b), however, you can see that the graph is not a line. So, when you are graphing an equation, either with or without a graphing utility, you must realize that different "viewing windows" can produce very different views of a graph. In choosing a viewing window, your goal is to show a view of the graph that fits well in the context of the problem.



Graphing utility screens of $y = x^3 - x^2 - 25$

Figure P.4

Exploration

Comparing Graphical and Analytic Approaches

Use a graphing utility to graph each equation. In each case, find a viewing window that shows the important characteristics of the graph.

a. $y = x^3 - 3x^2 + 2x + 5$

b. $y = x^3 - 3x^2 + 2x + 25$

c. $y = -x^3 - 3x^2 + 20x + 5$

d. $y = 3x^3 - 40x^2 + 50x - 45$

e. $y = -(x + 12)^3$

f. $y = (x - 2)(x - 4)(x - 6)$

A purely graphical approach to this problem would involve a simple "guess, check, and revise" strategy. What types of things do you think an analytic approach might involve? For instance, does the graph have symmetry? Does the graph have turns? If so, where are they? Later, in Chapters 1, 2, and 3, you will study many new analytic tools that will help you analyze graphs of equations.

*In this text, the term *graphing utility* refers to graphing calculators (such as the *TI-Nspire* and *Desmos*) and mathematical software (such as *Maple* and *Mathematica*).

REMARK Some texts denote the x-intercept as the x-coordinate of the point $(a, 0)$ rather than the point itself. Unless it is necessary to make a distinction, when the term *intercept* is used in this text, it will mean either the point or the coordinate.

Intercepts of a Graph

Two types of solution points that are especially useful in graphing an equation are those having zero as their x- or y-coordinate. Such points are called **intercepts** because they are the points at which the graph intersects the x- or y-axis. The point $(a, 0)$ is an ***x*-intercept** of the graph of an equation when it is a solution point of the equation. To find the x-intercepts of a graph, let y be zero and solve the equation for x. The point $(0, b)$ is a ***y*-intercept** of the graph of an equation when it is a solution point of the equation. To find the y-intercepts of a graph, let x be zero and solve the equation for y.

It is possible for a graph to have no intercepts, or it might have several. For instance, consider the four graphs shown in Figure P.5.

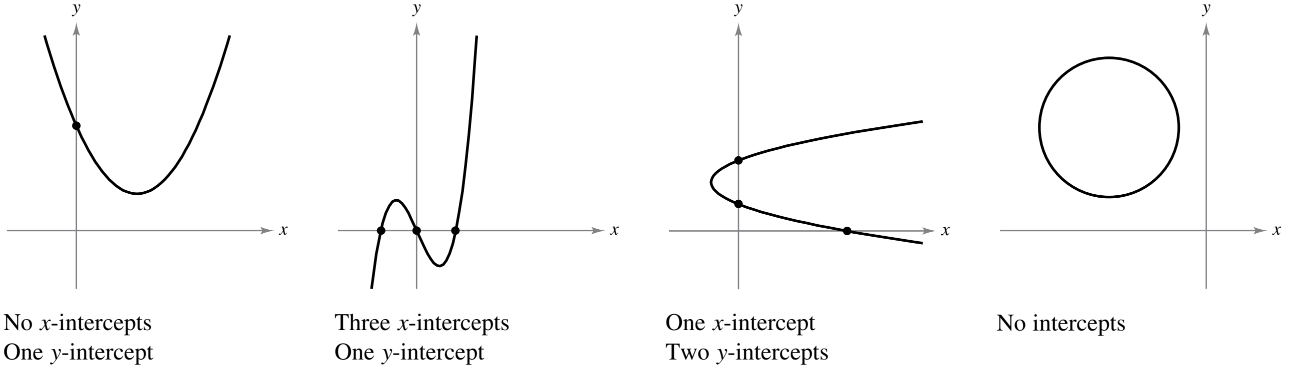


Figure P.5

EXAMPLE 2 Finding *x*- and *y*-Intercepts

Find the x- and y-intercepts of the graph of $y = x^3 - 4x$.

Solution To find the x-intercepts, let y be zero and solve for x.

$$x^3 - 4x = 0 \quad \text{Let } y \text{ be zero.}$$
$$x(x^2 - 4) = 0 \quad \text{Factor out common monomial factor.}$$
$$x(x - 2)(x + 2) = 0 \quad \text{Factor difference of two squares.}$$
$$x = 0, 2, \text{ or } -2 \quad \text{Solve for } x.$$

This equation has three solutions, so the graph has three x-intercepts:

$$(0, 0), \quad (2, 0), \quad \text{and} \quad (-2, 0). \quad x\text{-intercepts}$$

To find the y-intercepts, let x be zero. Doing this produces $y = 0$. So, the y-intercept is

$$(0, 0). \quad y\text{-intercept}$$

(See Figure P.6.)

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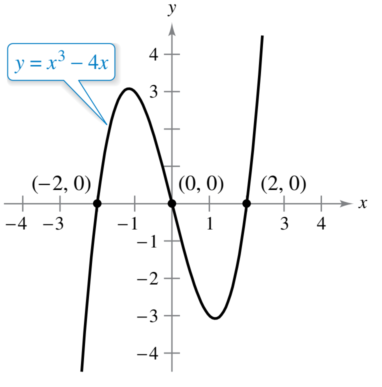


Intercepts of a graph
Figure P.6

TECHNOLOGY Example 2 uses an analytic approach to find the intercepts. When an analytic approach is not possible, use a graphical approach to find the points at which the graph intersects the axes. Use the *trace* feature of a graphing utility to approximate the intercepts of the graph of the equation in Example 2. Note that the utility may have a *root* or *zero* feature that can find the x-intercepts of a graph. If so, use this feature to find the x-intercepts of the graph of the equation in Example 2.

Symmetry of a Graph

Knowing the symmetry of a graph before attempting to sketch it is useful because you need only half as many points to sketch the graph. The three types of symmetry listed below can be used to help sketch the graphs of equations (see Figure P.7).

1. A graph is **symmetric with respect to the y-axis** if, whenever (x, y) is a point on the graph, then $(-x, y)$ is also a point on the graph. This means that the portion of the graph to the left of the y-axis is a mirror image of the portion to the right of the y-axis.
2. A graph is **symmetric with respect to the x-axis** if, whenever (x, y) is a point on the graph, then $(x, -y)$ is also a point on the graph. This means that the portion of the graph below the x-axis is a mirror image of the portion above the x-axis.
3. A graph is **symmetric with respect to the origin** if, whenever (x, y) is a point on the graph, then $(-x, -y)$ is also a point on the graph. This means that the graph is unchanged by a rotation of 180° about the origin.

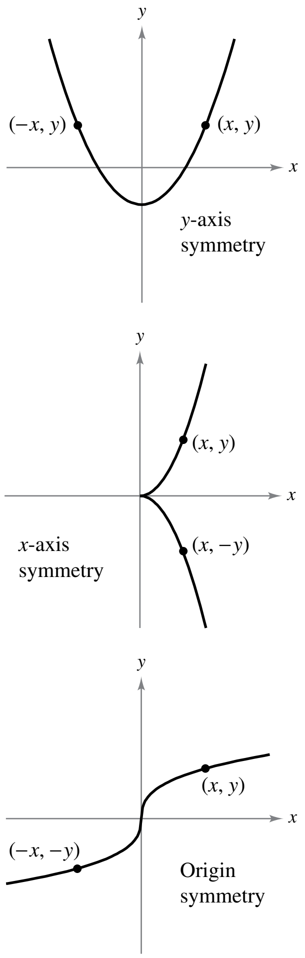


Figure P.7

Tests for Symmetry

1. The graph of an equation in x and y is symmetric with respect to the y-axis when replacing x by $-x$ yields an equivalent equation.
2. The graph of an equation in x and y is symmetric with respect to the x-axis when replacing y by $-y$ yields an equivalent equation.
3. The graph of an equation in x and y is symmetric with respect to the origin when replacing x by $-x$ and y by $-y$ yields an equivalent equation.

The graph of a polynomial has symmetry with respect to the y-axis when each term has an even exponent (or is a constant). For instance, the graph of

$$y = 2x^4 - x^2 + 2$$

has symmetry with respect to the y-axis. Similarly, the graph of a polynomial has symmetry with respect to the origin when each term has an odd exponent, as illustrated in Example 3.

GO DIGITAL

EXAMPLE 3 Testing for Symmetry

Test the graph of $y = 2x^3 - x$ for symmetry with respect to (a) the y-axis and (b) the origin.

Solution

a. $y = 2x^3 - x$ — Write original equation.

$y = 2(-x)^3 - (-x)$ — Replace x by $-x$.

$y = -2x^3 + x$ — Simplify. The result is *not* an equivalent equation.

Replacing x by $-x$ does *not* yield an equivalent equation, so the graph of $y = 2x^3 - x$ is *not* symmetric with respect to the y-axis.

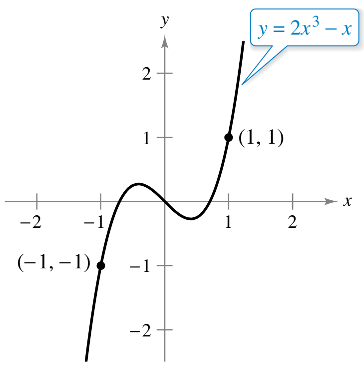
b. $y = 2x^3 - x$ — Write original equation.

$-y = 2(-x)^3 - (-x)$ — Replace x by $-x$ and y by $-y$.

$-y = -2x^3 + x$ — Simplify.

$y = 2x^3 - x$ — Equivalent equation

Replacing x by $-x$ and y by $-y$ yields an equivalent equation, so the graph of $y = 2x^3 - x$ is symmetric with respect to the origin, as shown in Figure P.8. ■



Origin symmetry
Figure P.8

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EXAMPLE 4 Using Intercepts and Symmetry to Sketch a Graph

See LarsonCalculus.com for an interactive version of this type of example.

Sketch the graph of $x - y^2 = 1$.

Solution The graph is symmetric with respect to the x-axis because replacing y by $-y$ yields an equivalent equation.

$$x - y^2 = 1 \quad \text{Write original equation.}$$
$$x - (-y)^2 = 1 \quad \text{Replace y by } -y.$$
$$x - y^2 = 1 \quad \text{Equivalent equation}$$

This means that the portion of the graph below the x-axis is a mirror image of the portion above the x-axis. To sketch the graph, first plot the x-intercept and the points above the x-axis. Then reflect in the x-axis to obtain the entire graph, as shown in Figure P.9.

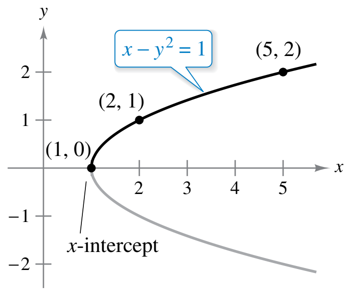


Figure P.9

TECHNOLOGY Some graphing utilities are designed so that they most easily graph equations in which y is a function of x (see Section P.3 for a definition of *function*). To graph other types of equations, you may need to split the graph into two or more parts *or* you may need to use a different graphing mode. For instance, one way to graph the equation in Example 4 is to split it into two parts.

$$y_1 = \sqrt{x - 1} \quad \text{Top portion of graph}$$
$$y_2 = -\sqrt{x - 1} \quad \text{Bottom portion of graph}$$

Points of Intersection

A **point of intersection** of the graphs of two equations is a point that satisfies both equations. You can find all points of intersection of two graphs by solving their equations simultaneously.

EXAMPLE 5 Finding Points of Intersection

Find all points of intersection of the graphs of

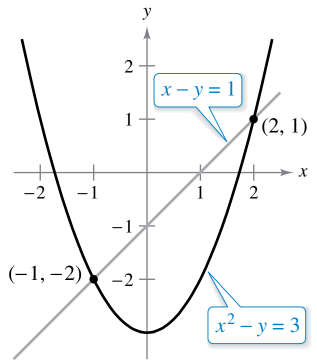
$$x^2 - y = 3 \quad \text{and} \quad x - y = 1.$$

Solution Begin by sketching the graphs of both equations in the *same* rectangular coordinate system, as shown in Figure P.10. From the figure, it appears that the graphs have two points of intersection. You can find these two points as follows.

$$y = x^2 - 3 \quad \text{Solve first equation for } y.$$
$$y = x - 1 \quad \text{Solve second equation for } y.$$
$$x^2 - 3 = x - 1 \quad \text{Equate } y\text{-values.}$$
$$x^2 - x - 2 = 0 \quad \text{Write in general form.}$$
$$(x - 2)(x + 1) = 0 \quad \text{Factor.}$$
$$x = 2 \text{ or } -1 \quad \text{Solve for } x.$$

The corresponding values of y are obtained by substituting $x = 2$ and $x = -1$ into either of the original equations. Doing this produces two points of intersection:

$$(2, 1) \quad \text{and} \quad (-1, -2). \quad \text{Points of intersection}$$



Two points of intersection
Figure P.10

You can check the points of intersection in Example 5 by substituting into *both* of the original equations or by using the *intersect* feature of a graphing utility.

Big Ideas of Calculus

You will use the concepts in this chapter throughout your study of calculus. Take the time to fully grasp each concept now so that you are ready to apply that concept later. Be sure to complete all concept exercises in this text—Concept Checks, Exploring Concepts, and Building on Concepts. These exercises are denoted by .

Mathematical Models

Real-life applications of mathematics often use equations as **mathematical models.** In developing a mathematical model to represent actual data, you should strive for two (often conflicting) goals—accuracy and simplicity. That is, you want the model to be simple enough to be workable, yet accurate enough to produce meaningful results. Appendix G explores these goals more completely.

EXAMPLE 6 Comparing Two Mathematical Models

The Mauna Loa Observatory in Hawaii records the carbon dioxide (CO_2) concentration y (in parts per million) in Earth's atmosphere. The January readings for various years are shown in Figure P.11. In 1990, these data were used to predict the carbon dioxide level in Earth's atmosphere in 2035, using the quadratic model

$$y = 0.020t^2 + 0.68t + 316.7 \quad \text{Quadratic model for 1960–1990 data}$$

where $t = 0$ represents 1960, as shown in Figure P.11(a). The data shown in Figure P.11(b) represent the years 1960 through 2020 and can be modeled by

$$y = 0.013t^2 + 0.85t + 316.2 \quad \text{Quadratic model for 1960–2020 data}$$

where $t = 0$ represents 1960. What was the prediction given by the first model for 1960 through 1990? Given the second model for the 1960 through 2020 data, does this prediction for 2035 seem accurate?

The Mauna Loa Observatory in Hawaii has been measuring the increasing concentration of carbon dioxide in Earth's atmosphere since 1958.

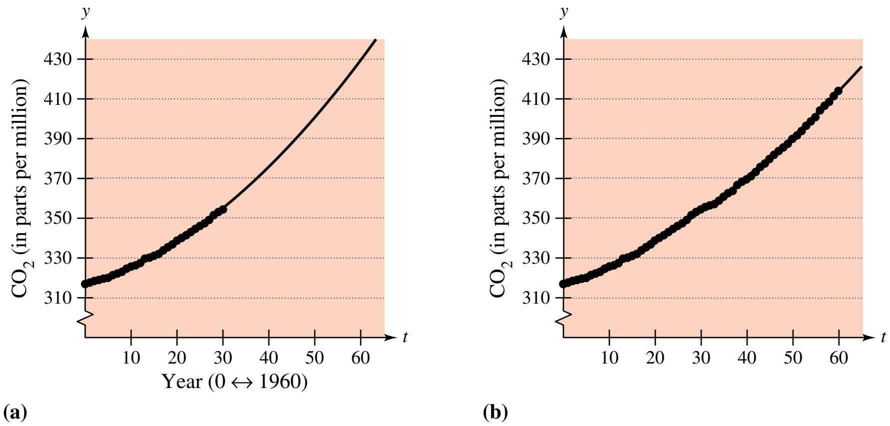


Figure P.11

Solution To answer the first question, substitute $t = 75$ (for 2035) into the first model.

$$y = 0.020(75)^2 + 0.68(75) + 316.7 = 480.2 \quad \text{Model for 1960–1990 data}$$

So, according to this model, the carbon dioxide concentration in Earth's atmosphere would reach about 480 parts per million in 2035. Using the model for the 1960–2020 data, the prediction for 2035 is

$$y = 0.013(75)^2 + 0.85(75) + 316.2 = 453.075. \quad \text{Model for 1960–2020 data}$$

So, based on the second model, it appears that the 1990 prediction was too high. ■

The models in Example 6 were developed using a procedure called *least squares regression* (see Section 13.9). Note that you can use the regression capabilities of a graphing utility to find a mathematical model (see Exercises 69 and 70).

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P.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

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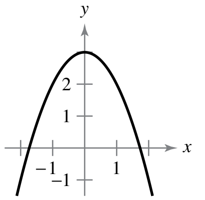
Concept Check

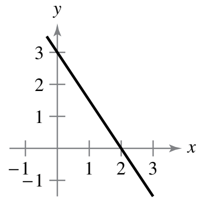
1. Describe how to find the x- and y-intercepts of the graph of an equation.
2. Explain how to use symmetry to sketch the graph of an equation.

Matching **In Exercises 3–6, match the equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]**

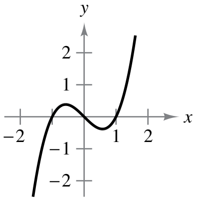
(a)



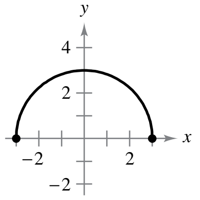
(b)



(c)



(d)



3. $y = -\frac{3}{2}x + 3$
4. $y = \sqrt{9 - x^2}$
5. $y = 3 - x^2$
6. $y = x^3 - x$

Sketching a Graph by Point Plotting **In Exercises 7–16, sketch the graph of the equation by point plotting.**

7. $y = \frac{1}{2}x + 2$
8. $y = 5 - 2x$
9. $y = 4 - x^2$
10. $y = (x - 3)^2$
11. $y = |x + 1|$
12. $y = |x| - 1$
13. $y = \sqrt{x} - 6$
14. $y = \sqrt{x + 2}$
15. $y = \dfrac{3}{x}$
16. $y = \dfrac{1}{x + 2}$

Approximating Solution Points **In Exercises 17 and 18, use a graphing utility to graph the equation. Move the cursor along the curve to approximate the unknown coordinate of each solution point accurate to two decimal places.**

17. $y = \sqrt{5 - x}$
 (a) $(2, y)$
 (b) $(x, 3)$
18. $y = x^5 - 5x$
 (a) $(-0.5, y)$
 (b) $(x, -4)$

Finding Intercepts **In Exercises 19–28, find any intercepts.**

19. $y = 2x - 5$
20. $y = 4x^2 + 3$
21. $y = x^2 + x - 2$
22. $y^2 = x^3 - 4x$
23. $y = x\sqrt{16 - x^2}$
24. $y = (x - 1)\sqrt{x^2 + 1}$
25. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$
26. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$
27. $x^2y - x^2 + 4y = 0$
28. $y = 2x - \sqrt{x^2 + 1}$

Testing for Symmetry **In Exercises 29–40, test for symmetry with respect to each axis and to the origin.**

29. $y = x^2 - 6$
30. $y = 9x - x^2$
31. $y^2 = x^3 - 8x$
32. $y = x^3 + x$
33. $xy = 4$
34. $xy^2 = -10$
35. $y = 4 - \sqrt{x + 3}$
36. $xy - \sqrt{4 - x^2} = 0$
37. $y = \dfrac{x}{x^2 + 1}$
38. $y = \dfrac{x^5}{4 - x^2}$
39. $y = |x^3 + x|$
40. $|y| - x = 3$

Using Intercepts and Symmetry to Sketch a Graph **In Exercises 41–58, find any intercepts and test for symmetry. Then sketch the graph of the equation.**

41. $y = 2 - 3x$
42. $y = \frac{2}{3}x + 1$
43. $y = 9 - x^2$
44. $y = 2x^2 + x$
45. $y = x^3 + 2$
46. $y = x^3 - 4x$
47. $y = x\sqrt{x + 5}$
48. $y = \sqrt{25 - x^2}$
49. $x = y^3$
50. $x = y^4 - 16$
51. $y = \dfrac{8}{x}$
52. $y = \dfrac{10}{x^2 + 1}$
53. $y = 6 - |x|$
54. $y = |6 - x|$
55. $x^2 + y^2 = 9$
56. $x^2 + 4y^2 = 4$
57. $3y^2 - x = 9$
58. $3x - 4y^2 = 8$

Finding Points of Intersection **In Exercises 59–64, find the points of intersection of the graphs of the equations.**

59. $x + y = 8$
 $4x - y = 7$
60. $3x - 2y = -4$
 $4x + 2y = -10$
61. $x^2 + y = 15$
 $-3x + y = 11$
62. $x = 3 - y^2$
 $y = x - 1$
63. $x^2 + y^2 = 5$
 $x - y = 1$
64. $x^2 + y^2 = 16$
 $x + 2y = 4$

A blue exercise number indicates that a video solution can be seen at *CalcView.com*.

The symbol indicates an exercise in which you are instructed to use a graphing utility or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by the use of appropriate technology.

Finding Points of Intersection **In Exercises 65–68, use a graphing utility to find the points of intersection of the graphs of the equations. Check your results analytically.**

65. $y = x^3 - 2x^2 + x - 1$
$y = -x^2 + 3x - 1$

66. $y = x^4 - 2x^2 + 1$
$y = 1 - x^2$

67. $y = \sqrt{x + 6}$
$y = \sqrt{-x^2 - 4x}$

68. $y = -|2x - 3| + 6$
$y = 6 - x$

69. Modeling Data The table shows the gross domestic product, or GDP (in trillions of dollars), for 2012 through 2019. *(Source: U.S. Bureau of Economic Analysis)*

Year	2012	2013	2014	2015	2016	2017	2018	2019
GDP	16.2	16.8	17.5	18.2	18.7	19.5	20.6	21.4

(a) Use the regression capabilities of a graphing utility to find a mathematical model of the form $y = at + b$ for the data. In the model, y represents the GDP (in trillions of dollars) and t represents the year, with $t = 12$ corresponding to 2012.

(b) Use a graphing utility to plot the data and graph the model. Compare the data with the model.

(c) Use the model to predict the GDP in the year 2029.

70. Modeling Data

The table shows the numbers of cell phone subscriptions (in billions) worldwide for 2012 through 2019.
(Source: Statista)

Year	2012	2013	2014	2015	2016	2017	2018	2019
Number	6.3	6.7	7.0	7.2	7.5	7.8	7.9	8.3

(a) Use the regression capabilities of a graphing utility to find a mathematical model of the form $y = at^2 + bt + c$ for the data. In the model, y represents the number of subscriptions (in billions) and t represents the year, with $t = 12$ corresponding to 2012.

(b) Use a graphing utility to plot the data and graph the model. Compare the data with the model.

(c) Use the model to predict the number of cell phone subscriptions worldwide in the year 2029.

71. Break-Even Point Find the sales necessary to break even $(R = C)$ when the cost C of producing x units is $C = 2.04x + 5600$ and the revenue R from selling x units is $R = 3.29x$.

72. Using Solution Points For what values of k does the graph of $y^2 = 4kx$ pass through the point?

(a) $(1, 1)$ (b) $(2, 4)$

(c) $(0, 0)$ (d) $(3, 3)$

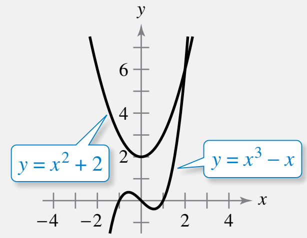
Exploring Concepts

73. Write an equation whose graph has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$. (There is more than one correct answer.)

74. A graph is symmetric with respect to the x-axis and to the y-axis. Is the graph also symmetric with respect to the origin? Explain.

75. A graph is symmetric with respect to one axis and to the origin. Is the graph also symmetric with respect to the other axis? Explain.

76. HOW DO YOU SEE IT? Use the graphs of the two equations to answer the questions below.



(a) What are the intercepts for each equation?

(b) Determine the symmetry for each equation.

(c) Determine the point of intersection of the two equations.

True or False? **In Exercises 77–80, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

77. If $(-4, -5)$ is a point on a graph that is symmetric with respect to the x-axis, then $(4, -5)$ is also a point on the graph.

78. If $(-4, -5)$ is a point on a graph that is symmetric with respect to the y-axis, then $(4, -5)$ is also a point on the graph.

79. If $b^2 - 4ac > 0$ and $a \neq 0$, then the graph of

$$y = ax^2 + bx + c$$

has two x-intercepts.

80. If $b^2 - 4ac = 0$ and $a \neq 0$, then the graph of

$$y = ax^2 + bx + c$$

has only one x-intercept.

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